A Novel Approach for Determining the GaAs MESFET Small-Signal Equivalent-Circuit Elements

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***Abstract*—A simple way of extracting the small-signal equivalent circuit of a MESFET is proposed. The intrinsic elements and one of the extrinsic parameters are described as functions of the remaining extrinsic parameters. This drastically reduces the search space and the number of unknowns for optimization. It provides an important new insight into the correlation between the various extrinsic and intrinsic parameters. The method also reveals that there are two sets of solution for the parameters R_s and L_s which can fully satisfy the global solution. A numerical comparison in terms of both accuracy and speed between the proposed method and some conventional methods on a 400-μm gatewidth GaAs MESFET is presented.**

I. Introduction

An ACCURATE characterization of the small-signal equivalent circuit is extremely important since it plays a crucial role in the extraction of the large-signal model. The typical method for the determination of the FET's equivalent circuit has been through the minimization of the difference between the measured and the computed S-parameters. However, this procedure may not produce unique element values for the equivalent circuit and the great number of variable elements produces an equivalent circuit strongly depending on the starting values. Moreover, the correlation between either the intrinsic parameters or one of the extrinsic parameters and the other extrinsic parameters is unclear. Only recently has Shirakawa [4] shown the dependence of the intrinsic parameters on the extrinsic parameters.

Numerous improvements on the characterization techniques have been proposed for GaAs MESFET [1]–[11], [13], but they commonly require extra measurements at pinchoff, dc, very low frequencies, and in the "cold state" to determine the extrinsic parameters. The determination of these extrinsic parameters is generally difficult, inaccurate, and time-consuming, and their bias dependence cannot be observed, even though theoretical considerations prove the existence of these dependencies [11], [12]. Thus, one can conclude that the key problem of obtaining a good physical equivalent circuit lies in the accurate determination of the extrinsic elements or, equivalently, in the reduction of the number of unknowns for optimization so that a smaller search space exists.

Manuscript received November 5, 1996; revised August 22, 1997.

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Publisher Item Identifier S 0018-9480(97)08241-0.

In this paper, we describe a technique to determine the MESFET equivalent circuit, which requires no additional measurements except the S-parameters. Our method is a fundamental step toward developing a nonlinear model for MESFET. This proposed method is an extension of [4] in that an additional equation is derived to further reduce the number of extrinsic parameters for optimization. As compared to Kondoh's [5] and Shirakawa's work [4], our search domain is now smaller in extent and only five unknowns are required for optimization. For the first time, the dependence of any one of the extrinsic parameters on the other extrinsic elements is shown. The derived expression shows that there exist two sets of solutions for both the elements R_s and L_s. A numerical comparison in terms of both accuracy and speed between the proposed method, the simulated annealing method [6], and the Kondoh's method [5] on a 400-μm gatewidth GaAs MESFET is presented.

II. Equivalent Circuit

A. The Intrinsic Elements

The adopted equivalent circuit is given in Fig. 1. As indicated in the figure, the intrinsic part of the device is enclosed by the dashed box, and the respective intrinsic Y-parameters are described as

$$Y_{\text{int}} = \begin{bmatrix} \dfrac{j\omega C_{\text{gs}}}{1+j\omega C_{\text{gs}}R_i} + j\omega C_{\text{dg}} & -j\omega C_{\text{dg}} \\ \dfrac{g_m e^{-j\omega\tau}}{1+j\omega C_{\text{gs}}R_i} - j\omega C_{\text{dg}} & g_{\text{ds}} + j\omega(C_{\text{dg}}+C_{\text{gs}}) \end{bmatrix}. \tag{1}$$

The extrinsic part of the device, which is located outside the dashed box of Fig. 1, is related to the intrinsic part through the following expression:

$$Y_{\text{int}} = (Z_{\text{total}} - Z_{\text{ext}})^{-1} \tag{2}$$

where Z_{total} is the overall Z-parameters and Z_{ext} is the extrinsic Z-parameters.

Once the extrinsic elements are known, along with the equivalent circuit as shown in Fig. 1, we can analytically determine the intrinsic elements. The overall measured S-parameters are first converted to Z-parameters, and by using (2), the intrinsic Y-parameters are obtained. The intrinsic elements at each frequency point has been derived in [4, eq.

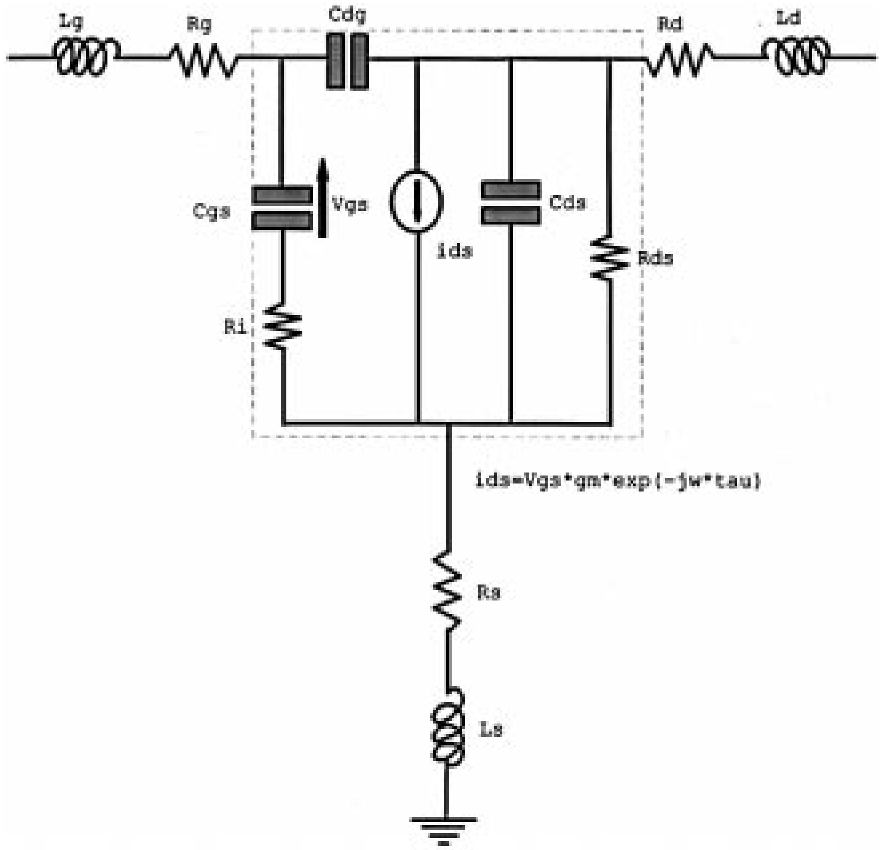

Fig. 1. Equivalent circuit of GaAs MESFET. The dashed box contained the intrinsic part of the device.

(4)–(12)], but for brevity, they are expressed below as a function of the remaining extrinsic elements. They are

$$C_{\mathrm{gs}} = f_1(\omega,\, R_s,\, R_d,\, L_g,\, L_s,\, L_d) \tag{3}$$

$$R_i = f_2(\omega,\, R_s,\, R_d,\, L_g,\, L_s,\, L_d) \tag{4}$$

$$C_{\mathrm{dg}} = f_3(\omega,\, R_s,\, R_d,\, L_g,\, L_s,\, L_d) \tag{5}$$

$$C_{\mathrm{ds}} = f_4(\omega,\, R_s,\, R_d,\, L_g,\, L_s,\, L_d) \tag{6}$$

$$g_{\mathrm{ds}} = f_5(\omega,\, R_s,\, R_d,\, L_g,\, L_s,\, L_d) \tag{7}$$

$$g_m = f_6(\omega,\, R_s,\, R_d,\, L_g,\, L_s,\, L_d) \tag{8}$$

$$\tau = f_7(\omega,\, R_s,\, R_d,\, L_g,\, L_s,\, L_d) \tag{9}$$

where ω is the angular frequency. With reference to (3)–(9), we use the function names f_0 to f_7 for convenience in the following discussion of our proposed method. Our method goes beyond what [4] has presented: an additional analytical equation has been derived to further improve the search space and reduce the number of optimization variables. The detailed derivation of this equation is provided in the Section II-B.

B. The Extrinsic Elements

Since the real part of the intrinsic matrix element Y_{12} is zero, an additional equation in terms of the extrinsic elements can be found using (2), which is reproduced here as

$$\begin{aligned}\mathrm{Re}\,([R_s + jwL_s - |Z_{12}|e^{\mathrm{jq}_{12}}]\\ *\,[(|Z_{11}|e^{-\mathrm{jq}_{11}} - R_g + jwL_g)\\ \cdot\,(|Z{\cdot}11|e^{-\mathrm{jq}_{11}} - R_d + jwL_d) - \xi]) = 0\end{aligned} \tag{10}$$

where

$$\begin{aligned}\xi = (R_s - j\omega L_s)(|Z_{11}|e^{-j\theta_{11}} + |Z_{22}|e^{-j\theta_{22}} - |Z_{12}|e^{-j\theta_{12}}\\ - |Z_{21}|e^{-j\theta_{21}} - \zeta) + |Z_{12}||Z_{21}|e^{-j(\theta_{12}+\theta_{21})}\end{aligned} \tag{11}$$

$$\zeta = R_g + R_d - j\omega(L_g + L_d) \tag{12}$$

and Z_{ij} with subscripts i, j being 1, 2 is the matrix element of the overall Z-parameters.

As described in the Section II-A, once the extrinsic elements are obtained, the intrinsic elements can be analytically determined. Conventional optimizer programs often assume that all the elements have the same accuracy and correlation, and that the extrinsic parameters can fluctuate widely about their initial values. At microwave frequencies, it is noted that with a slight change in any of the extrinsic elements' values, some drastic changes in some of the intrinsic elements' values are noted. For example, the frequency characteristic of C_{gs} with respect to R_s is plotted in Fig. 2(a). If the equivalent circuit is valid at every measured frequency, the frequency response of the element will be independent of frequency. As shown in Fig. 2(a), there exist large fluctuations in the frequency response of C_{gs} with changes in the values of R_s. The changes in the intrinsic elements' values with a small change in the extrinsic elements is solely governed by (2)–(10). In addition, (10) indicates that there is a strong correlation among the extrinsic elements' values such that they cannot assume wide initial values as they will cause some of the intrinsic elements' values to be negative or eventually a

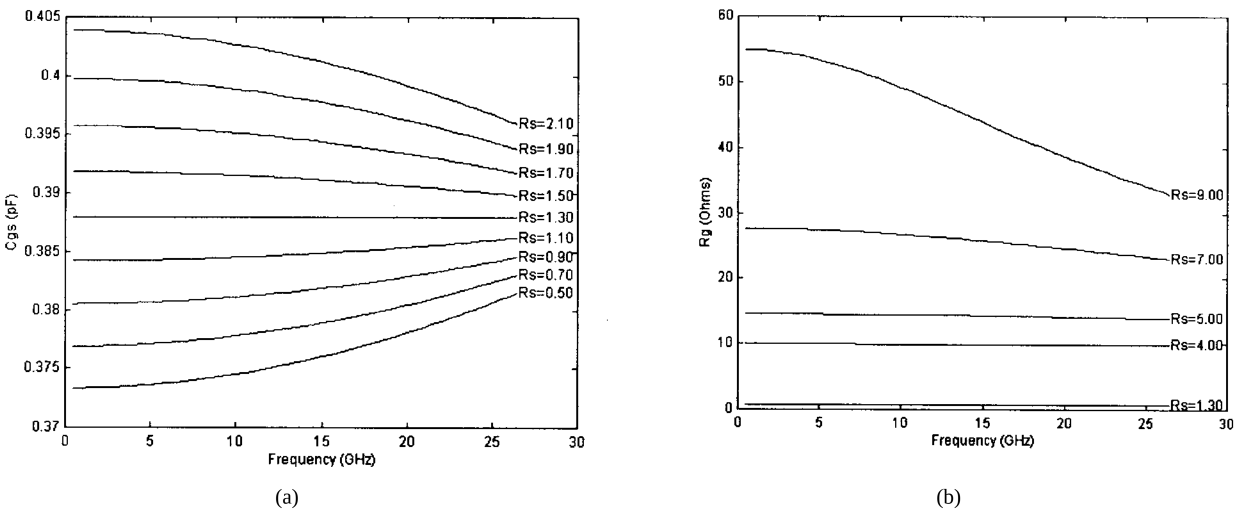

Fig. 2. (a) Frequency response of $C_{\rm gs}$ with R_s at $V_{\rm ds} = 8$ V, $I_{\rm ds} = 0.6 I_{\rm dss}$, gatewidth $= 400$ μm. (b) Frequency response of R_g with different R_s at $V_{\rm ds} = 8$ V, $I_{\rm ds} = 0.6$ $I_{\rm dss}$, gatewidth $= 400$ μm.

termination to a local minimum. Equivalently, a bad match between the measured and calculated S_{21} will be observed. Fig. 2(b) plots the frequency response of R_g with respect to R_s. As illustrated in the Fig. 2(b), a strong correlation between R_g and R_s is observed, since the frequency response of R_g is not independent of frequency. With regards to [7], if R_s or R_d is bias dependent, the nonlinear effect can also be reflected onto R_g through (10).

From (10), we also see that at least one of the extrinsic elements can be expressed in terms of the remaining extrinsic elements. For clarity and simplicity in demonstration, we choose to express R_g in terms of R_s, R_d, L_g, L_s, and L_d, i.e.,

$$R_g = f_0(\omega, R_s, R_d, L_g, L_s, L_d). \tag{13}$$

Other variations can be derived from (11) and will not be presented here for brevity. For clarity, only the expression for R_g is given in the Appendix. Equation (11) also reveals that there exist two sets of solutions for both R_s and L_s and, in general, the other solution of either R_s or L_s is usually negative in value.

Although Fig. 1 has been adopted for our analysis, (11) or (13) can be extended to some other equivalent circuits. As an example, by substituting

$$[Z] = \left(\begin{bmatrix} Y_{11}^{\rm Total} & Y_{12}^{\rm Total} \\ Y_{21}^{\rm Total} & Y_{22}^{\rm Total} \end{bmatrix} - \begin{bmatrix} j\omega C_{\rm pgs} & 0 \\ 0 & j\omega C_{\rm pds} \end{bmatrix} \right)^{-1} \tag{14}$$

into (4)–(10) and (13), the equivalent circuit with pad capacitances $C_{\rm pgs}$ and $C_{\rm pds}$ can be adopted. In here, the first term of the enclosed bracket of (14) is the total measured Y-parameters and $C_{\rm pgs}$ and $C_{\rm pds}$ are the two added pad capacitances.

III. The Algorithm

The conventional approach usually does not check circuit validity and determine extrinsic and intrinsic elements *simultaneously*. Extrinsic elements determined in this way often invalidate the circuit when the operating bias is changed, and often additional measurements performed at pinchoff, dc, low frequency, and cold state are required to narrow down the search [2], [3], [8], [9].

Our method is an extension of Shirakawa's approach [4] in that the intrinsic elements as well as one of the extrinsic elements are used as an optimization criterion. The first objective function for our approach is presented as

$$\begin{aligned} \varepsilon_1^k(R_s, R_d, L_g, L_s, L_d) &= \frac{1}{N-1} \sum_{i=0}^{N-1} \Bigg| \rho_k f_k(\omega_i, R_s, R_d, L_g, L_s, L_d) \\ &\quad - \overline{\sum_{j=0}^{N-1} \rho_k f_k(\omega_i, R_s, R_d, L_g, L_s, L_d)} \Bigg|^2 \end{aligned} \tag{15}$$

where k varies from zero to seven, N is the total number of frequency points, ρ_k is a normalizing factor to make f_k vary between zero and one, and the overbar indicates the mean value. For stable calculations, the discrepancy between the measured and calculated S-parameters must also be checked. For this reason, the second objective function is selected as

$$\begin{aligned} \varepsilon_2(R_s, R_d, L_g, L_s, L_d) &= \sum_{p=1}^{2} \sum_{q=1}^{2} \sum_{i=0}^{N-1} \sigma_{\rm pq} |S_{\rm pq}^m(\omega_i, R_s, R_d, L_g, L_s, L_d) \\ &\quad - \overline{S}_{\rm pq}^c(R_s, R_d, L_g, L_s, L_d)| \end{aligned} \tag{16}$$

where superscripts c and m denote, respectively, the calculated and measured S-parameters $\sigma_{\rm pq}$, which is arbitrarily selected at 0.5 and is the weighting factor, and the overbar indicates that the mean values of f_0 to f_7 are used to compute the S-parameters. Taking the mean values of f_0 to f_7 implies that the global solution occurs at the mean values of these functions. The extended error vector is then composed of

$$\varepsilon(R_s, R_d, L_g, L_s, L_d) = \begin{bmatrix} \varepsilon_1^k(R_s, R_d, L_g, L_s, L_d) \\ \varepsilon_2(R_s, R_d, L_g, L_s, L_d) \end{bmatrix}. \tag{17}$$

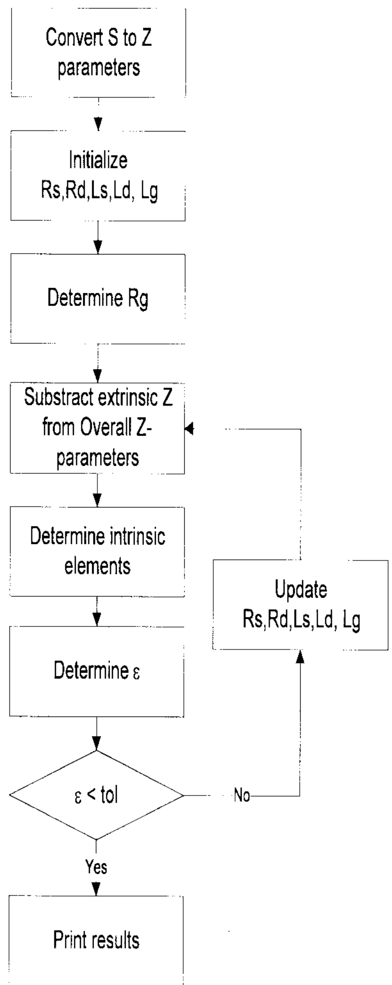

Fig. 3. The algorithm.

By selectively assigning values for the remaining extrinsic parameters, R_g, C_{gs}, R_i, C_{dg}, C_{ds}, g_{ds}, g_m, and τ can then be chronologically evaluated from (4) to (10) by iteration (13). A flowchart of the iterative process is shown in Fig. 4.

IV. Experiments, Results, and Discussions

A MATLAB program has been written based on the flowchart given in Fig. 3. The values of R_s, R_d, L_g, L_s, and L_d are initially generated by [13] and updated by the Levenberg–Marquart method.[1]

Table I gives the comparison of the error contributed from the difference between simulated and measured S-parameters of a 400-μm GaAs MESFET using the proposed method, Kondoh's method [5], and simulated annealing method [6]. The same initial guesses are applied for all these methods and the time taken for these methods are also presented in Table I.

As noted from Table I, the proposed method has the lowest error of about 1.45e-7 as compared to the Kondoh's method (9.39e-4) and the simulated annealing method (6.23e-5). In addition, the proposed method has a faster rate of convergence and takes about 1.2 min for completion. In our simulation, we have also observed that the validity of the circuit is guaranteed by the elements' independence of frequency. The comparison between the measured and calculated S-parameters under the proposed method for a 400-μm gatewidth GaAs MESFET is presented in Fig. 4. Close agreement between the measured and calculated S-parameters is noted from the figure.

Similar to [4], our method can also be extended to multibias extraction. As noted, once the unknowns R_s, R_d, L_g, L_s, and L_d are known at all biases, the corresponding iterative elements, i.e., of C_{gs}, C_{ds}, C_{dg}, R_i, g_m, τ, g_{ds}, and R_g at all bias points can be found. For brevity and clarity, the multibias extraction is not presented here.

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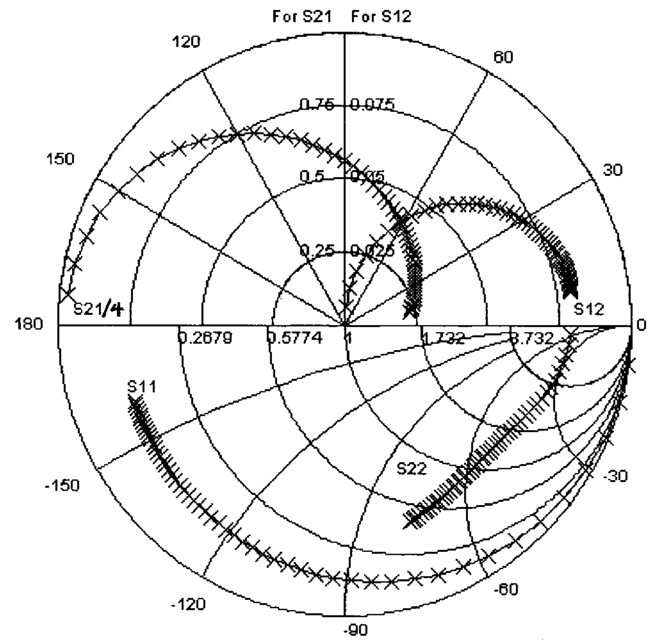

Fig. 4. S-parameters of 400-μm GaAs MESFET. Frequency: 0.5–26.5 GHz. Bias: $V_{ds} = 8$ V, $I_{ds} = 0.6\ I_{dss}$. Solid-line: Simulated values. Asterisk: Measured values.

TABLE I
A Comparison of the Time Taken Between the Proposed Method and the Conventional Methods

Items	Method from [5]	Classical Simulated Annealing Method	The Proposed Method
$\sum_{j=1}^{2}\sum_{i=1}^{2}\left\|S_{ij}^{cal} - S_{ij}^{meas}\right\|$	9.39e-4	6.23e-5	1.45e-7
Time Taken	8.65 min	45.23 min	1.2 min

V. Conclusion

A simple technique for the small-signal equivalent-circuit parameter extraction has been proposed. For the first time, an explicit equation which results in a reduction of the number of optimization variables is derived. In addition, a new insight into the correlation between the intrinsic and the extrinsic parameters or the correlation between the extrinsic parameters and one of the extrinsic elements is presented. This new method is found to be more accurate as compared to the conventional methods.

APPENDIX

The expression for R_g is given as

$$\begin{aligned} R_g = {} & [(R_s - Z_{12r})(\omega L_g\{Z_{22i} - \omega L_d - \omega L_s\} + R_d(R_s - Z_{11r}) \\ & - R_s(Z_{11r} - Z_{12r} - Z_{21r} + Z_{22r}) - \omega L_d(\omega L_s - Z_{11i}) \\ & + \omega L_s(Z_{11i} - Z_{12i} - Z_{21i} + Z_{22i}) - Z_{11i}Z_{22i} + Z_{12i}Z_{21i} \\ & - Z_{12r}Z_{21r} + Z_{22r}Z_{11r}) + (\omega L_S - z_{12i}) \\ & \cdot (\omega L_g(R_d + R_s - Z_{22r}) + R_d(\omega L_s - Z_{11i}) \\ & + R_s(\omega L_d - Z_{11i} + Z_{12i} + Z_{21i} - Z_{22i}) - \omega L_d Z_{11r} \\ & - \omega L_s(Z_{11r} - Z_{12r} - Z_{21r} + Z_{22r}) + Z_{11r}Z_{22i} \\ & - Z_{21i}Z_{12r} - Z_{12i}Z_{21r} + Z_{22r}Z_{11i})] \\ & \cdot [(R_s - Z_{12r})(R_d + R_s + Z_{22r}) + (\omega L_s - Z_{12i}) \\ & \cdot (\omega L_d + \omega L_s - Z_{22i})]^{-1} \end{aligned} \tag{18}$$

where subscripts i and r are, respectively, the real and imaginary terms.

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